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critical dynamics in a realistic seaway A new method to predict vessel/platform

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S. Vishnubhotla, J. Falzarano and A. Vakakis

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A new method to predict vessel/platform

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critical dynamics in a realistic seaway **critical dynamics in a realistic seaway**
BY S. VISHNUBHOTLA¹, J. FALZARANO¹ AND A. VAKAKIS²

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University of Illinois, IL 61801, USA

In this paper, a recently developed approach is used that makes use of a closed form
analytic solution, which is exact up to the first order of randomness, and takes into In this paper, a recently developed approach is used that makes use of a closed form
analytic solution, which is exact up to the first order of randomness, and takes into
account exactly the unperturbed (no forcing or dam In this paper, a recently developed approach is used that makes use of a closed form
analytic solution, which is exact up to the first order of randomness, and takes into
account exactly the unperturbed (no forcing or damp analytic solution, which is exact up to the first order of randomness, and takes into
account exactly the unperturbed (no forcing or damping) global dynamics. The result of this is that very-large-amplitude nonlinear vessel motion in a random seaway can
be analysed with techniques similar to those used to analyse nonlinear vessel motions
in a regular (periodic) seaway; the practical result be analysed with techniques similar to those used to analyse nonlinear vessel motions be analysed with techniques similar to those used to analyse nonlinear vessel motions
in a regular (periodic) seaway; the practical result being that dynamic capsizing stud-
ies can be undertaken considering the true rando in a regular (periodic) seaway; the practical result being that dynamic capsizing stud-
ies can be undertaken considering the true randomness of the design seaway. The
capsize risk associated with operation in a given sea capsize risk associated with operation in a given sea spectrum can be evaluated during the design stage or when an operating area change is being considered. Moreover, capsize risk associated with operation in a given sea spectrum can be evaluated during the design stage or when an operating area change is being considered. Moreover, this technique can also be used to guide physical mode ing the design stage or when an operating area change is being considered. Moreover,
this technique can also be used to guide physical model tests or computer simulation
studies to focus on critical vessel and environmenta this technique can also be used to guide physical model tests or computer simulation
studies to focus on critical vessel and environmental conditions which may result in
dangerously large motion amplitudes. In order to dem studies to focus on critical vessel and environmental conditions which may result in dangerously large motion amplitudes. In order to demonstrate the practical usefulness of this approach, extensive comparative results are dangerously large motion amplitudes. In order to demonstrate the practical useful-
ness of this approach, extensive comparative results are included. The results are
in the form of solutions which lie in the stable or unst ness of this approach, extensive
in the form of solutions which lie
projected onto the phase plane. projected onto the phase plane.
Keywords: nonlinear ship/platform motions; stochastic vessel dynamics;

critical ship/platform roll dynam ics; nonlinear dynam ical system s; phase plane

1. Introduction and background

Research studies of nonlinear ship and floating offshore platform rolling motion using dynamical systems' approaches have become quite common (Thompson 1997). How-Research studies of nonlinear ship and floating offshore platform rolling motion using
dynamical systems' approaches have become quite common (Thompson 1997). How-
ever, practical ship design stability criteria still focus dynamical systems' approaches have become quite common (Thompson 1997). How-
ever, practical ship design stability criteria still focus on the static restoring moment
curve as the sole or dominant indicator of the vessel's ever, practical ship design stability criteria still focus on the static restoring moment
curve as the sole or dominant indicator of the vessel's resistance to capsizing and
only consider the motion in an implicit or very Fourve as the sole or dominant indicator of the vessel's resistance to capsizing and \bigcup only consider the motion in an implicit or very approximate manner. Most nonlinear \bigcap motions studies are limited to single-deg only consider the motion in an implicit or very approximate manner. Most nonlinear
motions studies are limited to single-degree-of-freedom and regular-wave (periodic)
excitation, with few exceptions (see, for example, Hsie motions studies are limited to single-degree-of-freedom and regular-wave (periodic) excitation, with few exceptions (see, for example, Hsieh *et al.* 1993; Simiu & Frey 1996; Soliman & Thompson 1990; Lin & Yim 1995). It i excitation, with few exceptions (see, for example, Hsieh *et al.* 1993; Simiu & Frey 1996; Soliman & Thompson 1990; Lin & Yim 1995). It is well known that roll cannot always be decoupled from the other degrees of freedom, 1996; Soliman & Thompson 1990; Lin & Yim 1995). It is well known that roll cannot always be decoupled from the other degrees of freedom, but more importantly it is well known that sea waves are not regular but are in fact mot always be decoupled from the other degrees of freedom, but more importantly
it is well known that sea waves are not regular but are in fact random. It is com-
mon in the design of ships and floating offshore platforms it is well known that sea waves are not regular but are in fact random. It is com-
mon in the design of ships and floating offshore platforms to make narrow banded
assumptions and predict short-term extremes using the Rayl function (PDF) (see, for example, Ochi 1998). In this study, the highly nonlinear

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near-capsizing behaviour of a small fishing vessel and a very large semi-submersible platform in a random seaway is analysed by using an analytical solution to the difnear-capsizing behaviour of a small fishing vessel and a very large semi-submersible
platform in a random seaway is analysed by using an analytical solution to the dif-
ferential equation. The availability of such a closed platform in a random seaway is analysed by using an analytical solution to
ferential equation. The availability of such a closed form solution allows sa
boundary curves for this pseudo-randomly forced system to be generate Fential equation. The availability of such a closed form solution allows safe basin
undary curves for this pseudo-randomly forced system to be generated.
The *Patti-B* was a small fishing vessel which has the dubious dist

boundary curves for this pseudo-randomly forced system to be generated.
The *Patti-B* was a small fishing vessel which has the dubious distinction of having capsized twice. This vessel operated off the east coast of the U The *Patti-B* was a small fishing vessel which has the dubious distinction of having capsized twice. This vessel operated off the east coast of the United States and was involved in two capsizings. Initially, she capsized capsized twice. This vessel operated off the east coast of the United States and was involved in two capsizings. Initially, she capsized in shallow water and her owners salvaged her (NTSB 1979). The second time, the vessel involved in two capsizing
salvaged her (NTSB 1979
and all hands were lost.
The 'mobile offshore b lvaged her (NTSB 1979). The second time, the vessel capsized in deeper waters
d all hands were lost.
The 'mobile offshore base' (MOB) is a structure, approximately one mile long,
ade up of three to five individual, large,

and all hands were lost.
The 'mobile offshore base' (MOB) is a structure, approximately one mile long,
made up of three to five individual, large, semi-submersible single base units (SBUs).
Transit draft has been identifie The 'mobile offshore base' (MOB) is a structure, approximately one mile long,
made up of three to five individual, large, semi-submersible single base units (SBUs).
Transit draft has been identified as a particular area of made up of three to five individual, large, semi-submersible single base units (SBUs).
Transit draft has been identified as a particular area of concern from a dynamics and
stability standpoint. Although while at operating Transit draft has been identified as a particular area of concern from a dynamics and
stability standpoint. Although while at operating draft, semi-submersibles are rela-
tively transparent to wave excitation due to the ma stability standpoint. Although while at operating draft, semi-submersibles are rela-
tively transparent to wave excitation due to the majority of the hull volume being
submerged far below the water surface, while at transi tively transparent to wave excitation due to the majority of the hull volume being
submerged far below the water surface, while at transit draft, an individual uncon-
nected SBU operates essentially as a catamaran with a r submerged far below the water surface, while at transit draft, an individual uncon-
nected SBU operates essentially as a catamaran with a relatively high metacentric
height. With the lower hulls penetrating the water surfa nected SBU operates essentially as a catamaran with a relatively high metacentric
height. With the lower hulls penetrating the water surface, wave excitation can be
important. Moreover, due to the reduced freeboard of the height. With the lower hulls penetrating the water surface, wave excitation can be important. Moreover, due to the reduced freeboard of the lower hulls, wetting of the their tops may also occur. This will result in a param important. Moreover
their tops may also
considered herein.

2. Physical system modeling

2. Physical system modeling
The focus of this study is the highly nonlinear rolling motion of ships and floating
offshore platforms possibly leading to cansizing. For the small fishing yessel, the The focus of this study is the highly nonlinear rolling motion of ships and floating
offshore platforms, possibly leading to capsizing. For the small fishing vessel, the
roll axis is the critical motion axis. Even though s offshore platforms, possibly leading to capsizing. For the small fishing vessel, the roll axis is the critical motion axis. Even though semi-submersibles generally have critical dynamics about a diagonal axis (Kota *et al.* offshore platforms, possibly leading to capsizing. For the small fishing vessel, the roll axis is the critical motion axis. Even though semi-submersibles generally have roll axis is the critical motion axis. Even though semi-submersibles generally have
critical dynamics about a diagonal axis (Kota *et al.* 1997, 1998), due to the relatively
large length-to-beam ratio of the MOB SBUs, rol critical dynamics about a diagonal axis (Kota *et al.* 1997, 1998), due to the relatively large length-to-beam ratio of the MOB SBUs, roll is assumed to be critical for this analysis. Roll is, in general, coupled to the o **MATHEMATICAL,
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SCIENCES** large length-to-beam ratio of the MOB SBUs, roll is assumed to be critical for this analysis. Roll is, in general, coupled to the other degrees of freedom; however, under certain circumstances, it is possible to approximat analysis. Roll is, in general, coupled to the other degrees of freedom; however, under
certain circumstances, it is possible to approximately decouple roll from the other
degrees of freedom and to consider it in isolation. certain circumstances, it is possible to approximately decouple roll from the other
degrees of freedom and to consider it in isolation. This allows focus on the critical roll
dynamics. The decoupling is most valid for vess degrees of freedom and to consider it in isolation. This allows focus on the critical roll dynamics. The decoupling is most valid for vessels which are approximately fore–aft symmetric; this eliminates the yaw coupling. Th dynamics. The decoupling is most valid for vessels which are approximately fore–aft
symmetric; this eliminates the yaw coupling. The USNA generic MOB is exactly
fore and aft symmetric. Moreover, by choosing an appropriate symmetric; this eliminates the yaw coupling. The USNA generic MOB is exactly
fore and aft symmetric. Moreover, by choosing an appropriate roll-centre coordinate
system, the sway is approximately decoupled from the roll (We fore and aft symmetric. Moreover, by choosing an appropriate roll-centre coordinate system, the sway is approximately decoupled from the roll (Webster 1989). For ships, it has been shown in previous studies that even if th system, the sway is approximately decoupled from the roll (Webster 1989). For ships,
it has been shown in previous studies that even if the yaw and sway coupling are
included, the results differ only in a quantitative sen it has been shown in previous studies that even if the yaw and sway coupling are
included, the results differ only in a quantitative sense. The yaw and sway act as
passive coordinates and do not qualitatively affect the r cluded, the results differ only in a quantitative sense. The yaw and sway act as ssive coordinates and do not qualitatively affect the roll (Zhang & Falzarano 1994). The other issue is the modelling of the fluid forces ac

passive coordinates and do not qualitatively affect the roll (Zhang & Falzarano 1994).
The other issue is the modelling of the fluid forces acting on the hull. Generally speaking, the fluid forces are subdivided into excit The other issue is the modelling of the fluid forces acting on the hull. Generally speaking, the fluid forces are subdivided into excitations and reactions (Newman 1982). The wave-exciting force is composed of one part due THE
SOCI speaking, the fluid forces are subdivided into excitations and reactions (Newman 1982). The wave-exciting force is composed of one part due to incident waves and another due to the diffracted waves. These forces are strong 1982). The wave-exciting force is composed of one part due to incident waves and
another due to the diffracted waves. These forces are strongly a function of the
wavelength/frequency. The reactive forces are composed of hy another due to the diffracted waves. These forces are strongly a function of the wavelength/frequency. The reactive forces are composed of hydrostatic (restoring) and hydrodynamic reactions. The hydrostatics are most stron wavelength/frequency. The reactive forces are composed of hydrostatic (restoring)
and hydrodynamic reactions. The hydrostatics are most strongly nonlinear and are
calculated using a ship hydrostatics computer program. In o and hydrodynamic reactions. The hydrostatics are most strongly nonlinear and are calculated using a ship hydrostatics computer program. In order that the zeroth-
order solutions are expressed in terms of known analytic fun calculated using a ship hydrostatics computer program. In order that the zeroth-
order solutions are expressed in terms of known analytic functions, the restoring
moment curve needs to be fitted by a cubic polynomial. Alth order solutions are expressed in terms of known analytic functions, the restoring moment curve needs to be fitted by a cubic polynomial. Although this fit may be somewhat approximate for the MOB SBU case, it is assumed tha

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**MATHEMATICAL,
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history.

least the correct qualitative behaviour. It should be noted here that it is not much

more difficult to use a numerically generated zeroth-order solution which is based least the correct qualitative behaviour. It should be noted here that it is not much
more difficult to use a numerically generated zeroth-order solution which is based
upon an accurate higher-order righting-arm curve (Zha least the correct qualitative behaviour. It should be noted here that it is not much
more difficult to use a numerically generated zeroth-order solution which is based
upon an accurate higher-order righting-arm curve (Zhan more difficult to use a numerically generated zeroth-order solution which is based
upon an accurate higher-order righting-arm curve (Zhang & Falzarano 1994). The
hydrodynamic part of the reactive force is that due to the s upon an accurate higher-order righting-arm curve (Zhang & Falzarano 1994). The
hydrodynamic part of the reactive force is that due to the so-called radiated wave
force. The radiated wave force is subdivided into added mas hydrodynamic part of the reactive force is that due to the so-called radiated wave
force. The radiated wave force is subdivided into added mass (inertia) and radiated
wave damping. These two forces are also strongly a func Since the damping. These two forces are also strongly a function of frequency. However, \bigcirc since the damping is light, and for simplicity, constant values at a fixed frequency \bigcirc are assumed. Generally, an empirical **J** wave damping. These two forces are also strongly a function of frequency. However, since the damping is light, and for simplicity, constant values at a fixed frequency
are assumed. Generally, an empirically determined nonlinear viscous damping term
is included. However, such empirical viscous damping res are assumed. Generally, an empirically determine
is included. However, such empirical viscous dan
ship hulls. The resulting equation of motion is $\dot{\phi}| + \Delta GZ(\phi) = F(t).$ (2.1)

$$
(I_{44} + A_{44}(\omega_n))\ddot{\phi} + B_{44}(\omega_n)\dot{\phi} + B_{44q}\dot{\phi}|\dot{\phi}| + \Delta GZ(\phi) = F(t). \tag{2.1}
$$

The focus of this study is nonlinear ship and floating offshore platform rolling motion in a realistic seaway due to a pseudo-random wave excitation. The effect

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of seaway intensity is accurately considered. In order to obtain the roll-moment
excitation spectrum, the sea spectrum is multiplied by the roll-moment excitation of seaway intensity is accurately considered. In order to obtain the roll-moment excitation spectrum, the sea spectrum is multiplied by the roll-moment excitation response amplitude operator (RAO) squared (see equation of seaway intensity is accurately considered. In order to obtain the roll-moment excitation exponse amplitude operator (RAO) squared (see equation (2.2 a)). The RAO for the small fishing vessel in given in figure 1a. excitation spectrum, the sea spectrum is multiplied by the roll-moment excitation
response amplitude operator (RAO) squared (see equation $(2.2 a)$). The RAO for the
small fishing vessel in given in figure 1a.
The sea spec

small fishing vessel in given in figure $1a$.
The sea spectral model used for the small fishing vessel is t
(PM). The PM sea-state equation (Ochi 1998) is as follows:

te equation (Ochi 1998) is as follows:
\n
$$
S^+(\omega) = \frac{8.1 \times 10^{-3} g^2}{\omega^5} e^{-0.74(g/U_w \omega)^4},
$$

 $S^+(\omega) = \frac{8.1 \times 10^{-9} g}{\omega^5} e^{-0.74(g/U_w \omega)^4}$,
where U_w is the wind speed. The PM model is used for this case because it corre-
sponds to a fully developed seaway, which is in some sense the most severe. Moreover, where U_w is the wind speed. The PM model is used for this case because it corresponds to a fully developed seaway, which is in some sense the most severe. Moreover, the spectrum is a one-parameter spectrum so that solel where U_w is the wind speed. The PM model is used for this case because it corresponds to a fully developed seaway, which is in some sense the most severe. Moreover, the spectrum is a one-parameter spectrum so that solel sponds to a fully dev
the spectrum is a one
can be considered.
The sea spectral e spectrum is a one-parameter spectrum so that solely the effect of seaway intensity
n be considered.
The sea spectral model used for the SBU of the MOB is the NATO sea-state
scriptions, which use the Bretschneider sea-sta

can be considered.
The sea spectral model used for the SBU of the MOB is the NATO sea-state descriptions, which use the Bretschneider sea-state formula. The Bretschneider sea-
state formula (Ochi 1998) is expressed as The sea spectral model used for the SB
descriptions, which use the Bretschneider se
state formula (Ochi 1998) is expressed as state formula (Ochi 1998) is expressed as

$$
S^{+}(\omega) = 0.1687 H_{\rm s}^2 \frac{\omega_{\rm s}^4}{\omega^5} e^{-0.675 \omega_{\rm s}^4/\omega^4},
$$

 $S^+(\omega) = 0.1687 H_s^2 \frac{\omega_s}{\omega^5} e^{-0.675 \omega_s^4/\omega^4}$,
where H_s is the significant wave height (i.e. the average of the one-third highest
waves) and ω_s is the significant wave frequency. The NATO model is used because it where H_s is the significant wave height (i.e. the average of the one-third highest waves) and ω_s is the significant wave frequency. The NATO model is used because it corresponds to a typical random seaway encountered where H_s is the significant wave height (i.e. the average of the one-third highest waves) and ω_s is the significant wave frequency. The NATO model is used because it corresponds to a typical random seaway encountered waves) and ω_s is the significant wave frequency. The NATO model is used because it corresponds to a typical random seaway encountered in the North Atlantic operating areas of the NATO navies. In general, the Bretschnei corresponds to a typical random seaway encountered in the North Atlantic operating areas of the NATO navies. In general, the Bretschneider sea spectrum is a twoareas of the NATO navies. In general, the Bretschneider sea spectrum is a two-
parameter seaway with significant wave height and significant period as the two
parameters. However, using the NATO wave data, the seaway inten parameter seaway with significant wave
parameters. However, using the NATO v
number becomes the single parameter.
These two sea spectral formulae are b parameters. However, using the NATO wave data, the seaway intensity or sea-state
number becomes the single parameter.
These two sea spectral formulae are based upon limited theoretical analysis and

extensive wave data analysis. Various additional relationships can be derived by manipulating these formulae and applying the Rayleigh PDF (Ochi 1998).

Figure 1b; c shows the excitation spectra for the *Patti-B* and the corresponding time history of the forcing (in non-dimensional form) for a wind speed of U_{w} = Figure 1b, c shows the excitation spectra for the *Patti-B* and the corresponding
time history of the forcing (in non-dimensional form) for a wind speed of $U_w =$
2.75 m s⁻¹. Figure 2 is for larger U_w . The significant time history of the forcing (in non-dimensional form) for a wind speed of $U_w = 2.75 \text{ m s}^{-1}$. Figure 2 is for larger U_w . The significant wave heights for the sea spectra used for the *Patti-B* range from less than 0.6 2.75 m s⁻¹. Figure 2 is for larger U_w . The significant wave heights for the sea spectra used for the *Patti-B* range from less than 0.6 m to almost 2.3 m. Although a different sea-state formula is used for the MOB, in used for the *Patti-B* range from less than 0.6 m to almost 2.3 m. Although a different sea-state formula is used for the MOB, in order to relate the two vessels, the sea-state sea-state formula is used for the MOB, in order to relate the two vessels, the sea-state
intensity ranges from about sea state 1 to 4 (Bhattachrayya 1978) for the *Patti-B*
and sea states 5 to 9 for the MOB. Alternatively intensity ranges from about sea state 1 to 4 (Bhattachrayya 1978) for the
and sea states 5 to 9 for the MOB. Alternatively, the seaways considered
MOB have significant wave heights, which range from *ca*. 3.2 m to 13.8 m.
 and sea states 5 to 9 for the MOB. Alternatively, the seaways considered for the MOB have significant wave heights, which range from $ca.3.2$ m to 13.8 m. It should also be noted herein that the complicated MOB roll-respon

MOB have significant wave heights, which range from $ca.3.2$ m to 13.8 m.
It should also be noted herein that the complicated MOB roll-response RAO calculated was approximated by the smooth curve depicted in figure $3a$. It should also be noted herein that the complicated MOB roll-response RAO calculated was approximated by the smooth curve depicted in figure $3a$. However, the true RAO calculated exhibited numerous humps and hollows at h culated was approximated by the smooth curve depicted in figure 3a. However, the
true RAO calculated exhibited numerous humps and hollows at higher frequencies
beyond the peak. Due to the small amount of wave energy at the true RAO calculated exhibited numerous humps and hollows at higher frequencies
beyond the peak. Due to the small amount of wave energy at these frequencies, these
oscillations were ignored. The resulting excitation spectru \bigcirc beyond the peak. Due to the small amount of wave energy at these frequencies, these \bigcirc oscillations were ignored. The resulting excitation spectrum is decomposed into periodic components with random phase angles. oscillations were ignored. The resulting excitation spectrum is decomposed into periwould then assume the form shown in figure $3c$:

$$
S_{\mathcal{R}}^{+}(\omega) = |\mathcal{R}\mathcal{A}\mathcal{O}|^{2} S^{+}(\omega), \qquad (2.2 a)
$$

$$
S_{\mathcal{R}}^{\mathcal{N}}(\omega) = |\mathcal{R}\mathcal{A}\mathcal{O}|^{2} S^{+}(\omega),
$$
\n
$$
F(t) = \sum_{i=1}^{N} F_{\mathcal{M}}(\omega_{i}) \cos(\omega_{i} t + \gamma_{i}),
$$
\n(2.2 *b*)

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(b) *Patti-B* large-amplitude roll-moment excitation spectrum $(U_w = 10.0 \text{ m s}^{-1})$.

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$$
F_{\rm M}(\omega_i) = \sqrt{2S_{\rm R}^+(\omega_i)\Delta\omega}.
$$
 (2.2 c)

Figure 3b, c shows the MOB SBU excitation spectra and the corresponding time-
dependent force (in non-dimensional form) for a NATO sea state 5. Figure 4 is the
MOB excitation spectrum for sea state 9 Figure $3b, c$ shows the MOB SBU excitation spectra and the corresponding time-Figure $3b$, c shows the MOB SBU excitation dependent force (in non-dimensional form)
MOB excitation spectrum for sea state 9 . $\rm MOB$ excitation spectrum for sea state 9.
3. The dynamical perturbation method 囸

3. The dynamical perturbation method
The focus of this investigation is the extension of an approach previously used
to study the nonlinear dynamics of a small fishing yessel and a very large semi-The focus of this investigation is the extension of an approach previously used
to study the nonlinear dynamics of a small fishing vessel and a very large semi-
submersible platform due to pseudo-random wave excitation (Vi The focus of this investigation is the extension of an approach previously used
to study the nonlinear dynamics of a small fishing vessel and a very large semi-
submersible platform due to pseudo-random wave excitation (Vi Submersible platform due to pseudo-random wave excitation (Vishnubhotla *et al.* 1998, 1999). The approach was originally developed by Vakakis (1993, 1994) to calculate in closed form the homoclinic manifolds due to rapid 1998, 1999). The approach was originally developed by Vakakis (1993, 1994) to calculate in closed form the homoclinic manifolds due to rapidly varying periodic \bigcirc excitation. That approach was generalized to calculate calculate in closed form the homoclinic manifolds due to rapidly varying periodic excitation. That approach was generalized to calculate heteroclinic manifolds due
to pseudo-random wave excitation. Considering that random excitation is a realis-
tic model for ship and floating offshore platform motions to pseudo-random wave excitation. Considering that random excitation is a realistic model for ship and floating offshore platform motions at sea, this method was extended and then applied to consider the case of perturbed tic model for ship and floating offshore platform motions at sea, this method was
extended and then applied to consider the case of perturbed heteroclinic manifolds
due to an external excitation as approximated by a finite extended and then applied to consider the case of perturbed heteroclinic manifolds
due to an external excitation as approximated by a finite summation of regular
(periodic) wave components.

The solution to equations such as (2.1) with softening spring characteristics ex-(periodic) wave components.
The solution to equations such as (2.1) with softening spring characteristics ex-
hibits two greatly different types of motions depending upon the amplitude of the
forcing For small forcing a The solution to equations such as (2.1) with softening spring characteristics exhibits two greatly different types of motions depending upon the amplitude of the forcing. For small forcing amplitude, the first type of m *Phil. Trans. R. Soc. Lond.* A (2000)

¹⁹⁷² *S. Vishn[ubhotla, J. Falzarano and](http://rsta.royalsocietypublishing.org/) A. Vakakis* Downloaded from rsta.royalsocietypublishing.org

Figure 3. (a) MOB roll-moment excitation transfer function (RAO) (smoothed). (b) MOB
roll-moment excitation spectra (NATO sea state 5) for $[H, T_0] = [10 \ 7 \ 9 \ 7]$ (c) MOB corre-Figure 3. (a) MOB roll-moment excitation transfer function (RAO) (smoothed). (b) MOB roll-moment excitation spectra (NATO sea state 5) for $[H_s, T_0] = [10.7, 9.7]$. (c) MOB corre-sponding roll-moment excitation time history. roll-moment excitation spectra (NATO sea state 5) for $[H_s, T_0] = [10.7, 9.7]$. (c) MOB corresponding roll-moment excitation time history. Roll-moment excitation force (non-dimensionalized) for $[H_s, T_0] = [10.7, 9.7]$.

ized) for $[H_s, T_0] = [10.7, 9.7]$.
which is bounded and well behaved. For large amplitudes of forcing, the motion can
be such that a unidirectional rotation occurs. The boundary between these two types which is bounded and well behaved. For large amplitudes of forcing, the motion can
be such that a unidirectional rotation occurs. The boundary between these two types
of motions is called in the terminology of poplinear vi which is bounded and well behaved. For large amplitudes of forcing, the motion can
be such that a unidirectional rotation occurs. The boundary between these two types
of motions is called, in the terminology of nonlinear v be such that a unidirectional rotation occurs. The boundary between these two types of motions is called, in the terminology of nonlinear vibrations, the separatrix. This curve literally separates the two qualitatively dif of motions is called, in the terminology of nonlinear vibrations, the separatrix. This
curve literally separates the two qualitatively different motions. In the language of
nonlinear dynamical systems, these curves are cal curve literally separates the two qualitatively different motions. In the language of nonlinear dynamical systems, these curves are called the (upper and lower) saddle connections. The saddles are connected as long as no d nonlinear dynamical systems, these curves are called the (upper and lower) saddle
connections. The saddles are connected as long as no damping and forcing are con-
sidered in the system. Once damping is added to the system connections. The saddles are connected as long as no damping and forcing are considered in the system. Once damping is added to the system, the saddle connection breaks into stable and unstable manifolds. The stable manifo sidered in the system. Once damping is added to the system, the saddle connection
breaks into stable and unstable manifolds. The stable manifolds are most impor-
tant because they form the basin boundary between initial co breaks into stable and unstable manifolds. The stable manifolds are most important because they form the basin boundary between initial conditions which remain bounded and those that become unbounded. When periodic forcing tant because they form the basin boundary between initial conditions which remain
bounded and those that become unbounded. When periodic forcing is added to the
system, these manifolds oscillate periodically with time but *Phil. Trans. R. Soc. Lond.* A (2000)

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Figure 4. MOB large-amplitude roll-moment spectrum (NATO sea state 9). Roll-moment excitation spectra for $[H_s, T_0] = [45.9, 20.0]$.

Figure 4. MOD harge-amphetic for-moment spectrum (WITO sea state 5).
Roll-moment excitation spectra for $[H_s, T_0] = [45.9, 20.0]$.
Configuration after one period of the forcing. This forcing period is chosen for the
Poincaré configuration after one period of the forcing. This forcing period is chosen for the
Poincaré sampling time of such a periodic system. However, no such obvious Poincaré
time sampling exists for the pseudo-randomly forced s configuration after one period of the forcing. This forcing period is chosen f
Poincaré sampling time of such a periodic system. However, no such obvious Po
time sampling exists for the pseudo-randomly forced system studie incaré sampling time of such a periodic system. However, no such obvious Poincaré
ne sampling exists for the pseudo-randomly forced system studied herein.
In this investigation, the random wave forcing is approximated by a

time sampling exists for the pseudo-randomly forced system studied herein.
In this investigation, the random wave forcing is approximated by a summation of periodic components with random relative phase angles. Although th In this investigation, the random wave forcing is approximated by a summation of
periodic components with random relative phase angles. Although this representation
approximates the true random excitation as $N \to \infty$ and periodic components with random relative phase angles. Although this representation
approximates the true random excitation as $N \to \infty$ and $\Delta \omega \to 0$, for finite N
this does not occur. Actually, the 'random' signal repea this does not occur. Actually, the 'random' signal repeats itself after $T_R = 2\pi/\Delta\omega$.
Another relevant time period is the average or zero crossing period T_0 . Assuming the spectra is narrow banded, this might also be a Another relevant time period is the average or zero crossing period T_0 . Assuming the spectra is narrow banded, this might also be a good reference period for a Poincaré map. In lieu of Poincaré maps, we choose to trace spectra is narrow banded, this might also be a good reference period for a Poincaré map. In lieu of Poincaré maps, we choose to trace out single solution paths which are contained in the stable manifolds (see figure 5). Th map. In lieu of Poincaré maps, we choose to trace out single solution paths which
are contained in the stable manifolds (see figure 5). These are then projected onto
the phase plane.

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VCES the phase plane.
The solutions lying in the stable manifolds are calculated using the new approach.
This method is a perturbation method which begins with the undamped and unforced
separatrix (see figure 5) which for a si The solutions lying in the stable manifolds are calculated using the new approach.
This method is a perturbation method which begins with the undamped and unforced
separatrix (see figure 5) which, for a simple softening s The solutions lying in the stable manifolds are calculated using the new approach. This method is a peri
separatrix (see figure
in closed form, i.e. separatrix (see figure 5) which, for a simple softening spring (equation (3.1)), is known

$$
\ddot{x} + x - kx^3 = 0,\t\t(3.1)
$$

$$
\ddot{x} + x - kx^3 = 0,
$$
\n
$$
x(\tau) = \frac{1}{\sqrt{k}} \tanh\left(\frac{\tau - \tau_0}{\sqrt{2}} + \frac{1}{2}\right),
$$
\n
$$
(3.2 a)
$$

$$
x(t) = \frac{1}{\sqrt{k}} \tanh\left(\frac{\sqrt{2}}{\sqrt{2}} + \frac{1}{2}\right),
$$
\n
$$
\dot{x}(\tau) = \frac{1}{\sqrt{2k}} \operatorname{sech}^{2}\left(\frac{\tau - \tau_{0}}{\sqrt{2}} + \frac{1}{2}\right),
$$
\n(3.2*b*)

 $\dot{x}(\tau) = \frac{1}{\sqrt{2k}} \operatorname{sech}^2\left(\frac{\tau - \tau_0}{\sqrt{2}} + \frac{1}{2}\right),$ (3.2 b)
where τ is the scaled time and τ_0 is the scaled initial time. The first-order solution is
determined by using the method of variation of parameters where τ is the scaled time and τ_0 is the scaled initial time. The first-order solution is
determined by using the method of variation of parameters. Equation (2.1) can now
be scaled into the following form: where τ is the scaled time and τ_0 is
determined by using the method of
be scaled into the following form:

$$
\ddot{x} + x - kx^3 = \epsilon(-\gamma \dot{x} - \gamma_q \dot{x}|\dot{x}| + F(\eta)), \qquad (3.3)
$$

 $\ddot{x} + x - kx^3 = \epsilon(-\gamma \dot{x} - \gamma q \dot{x}|\dot{x}| + F(\eta)),$ (3.3)

where $\eta = (\tau - \tau_0)/\sqrt{2 + \frac{1}{2}}$. The solution method involves expanding the solution in

a perturbation series as where $\eta = (\tau - \tau_0)/\sqrt{2} + \frac{1}{2}$
a perturbation series as The solution method involves expanding the solution in
 $x(\eta) = x_0(\eta) + \epsilon x_1(\eta) + \cdots$ (3.4)

$$
x(\eta) = x_0(\eta) + \epsilon x_1(\eta) + \cdots. \tag{3.4}
$$

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Figure 5. Extended phase space showing stable manifold and contained time-varying solution.

The first-order equation to be solved is actually a linear equation with time vary-The first-order equation to be solved is actually a linear equation with time vary-
ing coefficients. The coefficients are obtained from the zeroth-order solution known
from $(3\ 1)$ and $(3\ 2)$ i.e The first-order equation
ing coefficients. The coeffic
from (3.1) and (3.2) , i.e. $\hat{G} = \hat{G}(x_0, \eta, \tau_0).$ (3.5)

$$
\ddot{x}_1 + x_1 - 3kx_1x_0^2 = \hat{G}(x_0, \eta, \tau_0). \tag{3.5}
$$

Once the homogeneous solution is determined, the particular solution is then Once the homogeneous solution is determined, the particular solution is then
obtained by again using the technique of variation of parameters. The above first-
order differential equation (3.5) is linear with non-consta Once the homogeneous solution is determined, the particular solution is then obtained by again using the technique of variation of parameters. The above first-order differential equation (3.5) is linear with non-constant obtained by again using the technique of variation of parameters. The above first-
order differential equation (3.5) is linear with non-constant coefficients and an inho-
mogeneous right-hand side \hat{G} . The non-constan order differential equation (3.5) is linear with non-constant coefficients and an inhomogeneous right-hand side \hat{G} . The non-constant coefficients and the right-hand side are composed of functions involving the now kn mogeneous right-hand side G . The non-constant coefficients and the right-hand side
are composed of functions involving the now known zeroth-order solution and other
known functions of time. The method is then used to de are composed of functions involving the now known zeroth-order solution and other
known functions of time. The method is then used to determine the critical solutions
which separate the bounded steady-state oscillatory mot known functions of time. The method is then used to determine the critical solutions
which separate the bounded steady-state oscillatory motions from the unbounded
motions. These solutions are determined by placing conditi which separate the bounded steady-state oscillate
motions. These solutions are determined by placing
eters which multiply the homogeneous solutions.
The details of the solution procedure can be sum to the solutions are determined by placing conditions on the varied parameters which multiply the homogeneous solutions.
The details of the solution procedure can be summarized as follows. To start, note at one of the fir

eters which multiply the homogeneous solutions.
The details of the solution procedure can be summarize
that one of the first-order homogeneous solutions, $x_{h1}^{(1)}$,
of the zeroth-order solution. The other first-order ho (1) : The arized as follows. To start, note
 $\begin{pmatrix} 1 \\ h_1 \end{pmatrix}$, is just the time derivative

nonogeneous solution $r^{(2)}$ can The details of the solution procedure can be summarized as follows. To start, note
that one of the first-order homogeneous solutions, $x_{h1}^{(1)}$, is just the time derivative
of the zeroth-order solution. The other first- (2) that one of the first-order homogeneous solutions, $x_{h1}^{(1)}$, is just the time derivative
of the zeroth-order solution. The other first-order homogeneous solution, $x_{h1}^{(2)}$, can
then be obtained by applying the metho of the zeroth-order solution. The other first-order homogeneous solution, $x_{h1}^{(2)}$, can
then be obtained by applying the method of variation of parameters and using the
Wronskian determinant. Once both homogeneous solu then be obtained by applying the method of variation of parameters and using the Wronskian determinant. Once both homogeneous solutions are known, the particular solutions can be obtained by again applying the method of va Wronskian determinant. Once both homogeneous solutions are known, the particular solutions can be obtained by again applying the method of variation of parameters. The total solution given in (3.6) is then attainable fr solutions can be obtained by again applying the method of variation of parameters.
The total solution given in (3.6) is then attainable from the summation of the homogeneous and particular parts. The only aspect that rema The total solution given in (3.6) is then attainable from the summation of the homogeneous and particular parts. The only aspect that remains is to determine the values of the variation constants, α and β . These val of the variation constants, α and β . These values are determined such that the stable
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solution remains bounded for all positive time to infinity while the unstable solution remains bounded for all negative time to infinity.

Vakakis originally developed this technique to study the Holmes–Duffing (buckled remains bounded for all negative time to infinity.
Vakakis originally developed this technique to study the Holmes–Duffing (buckled
beam) oscillator with a rapidly varying external excitation and the details of that
analys Vakakis originally developed this technique to study the Holmes–Duffing (buckled
beam) oscillator with a rapidly varying external excitation and the details of that
analysis are described in Vakakis (1993). The approach ta beam) oscillator with a rapidly varying external excitation and the details of that
analysis are described in Vakakis (1993). The approach taken in this paper, although
different from the original analysis, is similar enou analysis are described in Vakakis (1993). The approach taken in this paper, although
different from the original analysis, is similar enough that all the details need not be
completely repeated herein. The first-order solu completely repeated herein. The first-order solution for the perturbed manifolds is
as follows:

follows:
\n
$$
x_{1}(\eta) = x_{h1}^{(1)}(\eta) \left(\alpha - \int_{0}^{\eta} x_{h1}^{(2)}(\tau) \hat{G}(\tau) d\tau \right) + x_{h1}^{(2)}(\eta) \left(\beta - \int_{0}^{\eta} x_{h1}^{(1)}(\tau) \hat{G}(\tau) d\tau \right).
$$
\n(3.6)

The associated constants (the varied parameters) are determined such that the criti-
cal basin boundary solutions remain bounded for infinite time (Vakakis 1993). These The associated constants (the varied parameters) are determined such that the critical basin boundary solutions remain bounded for infinite time (Vakakis 1993). These conditions provide the stable and unstable manifolds a The associated constants (the varied parameters) are determined such that the critical basin boundary solutions remain bounded for infinite time (Vakakis 1993). These conditions provide the stable and unstable manifolds as cal basin boundary solutions remain bounded for infinite time (Vakakis 1993). These conditions provide the stable and unstable manifolds associated with the positive and negative angles of vanishing stability and correspon conditions provide the stable and unstable manifolds associated with the positive
and negative angles of vanishing stability and correspond to the upper and lower
separatrices, respectively. The stable manifolds form the b and negative angles of vanishing stability and correspond to the upper and lower
separatrices, respectively. The stable manifolds form the basin boundary between
bounded (safe non-capsizing) and unbounded (capsizing) solut separatrices, respectively. The stable manifolds form the basin boundary between
bounded (safe non-capsizing) and unbounded (capsizing) solutions (see, for exam-
ple, figure 5). It is also possible to determine the unstabl bounded (safe non-capsizing) and unbounded (capsizing) solutions (see, for example, figure 5). It is also possible to determine the unstable manifolds by choosing the variation constants such that the manifolds remain boun ple, figure 5). It is also possible to determine the unstable manifolds by choosing the variation constants such that the manifolds remain bounded for minus infinite time.
The unstable manifolds are important because they variation constants such that the manifolds remain bounded for minus infinite time.
The unstable manifolds are important because they interact with the stable manifolds and erode the safe basin. The integrals in (3.6) can The unstable manifolds are important because they interact with the stable manifolds and erode the safe basin. The integrals in (3.6) can be determined analytically in series form for simple springs and numerically for folds and erode the safe basin. The integrals in (3.6) can be determined analytically in series form for simple springs and numerically for more complicated springs. This is the key aspect of the method in that it is capable of yielding exact series solutions
for simple springs which later can be used to verify approximate numerical results.
Although this method was originally developed by

for simple springs which later can be used to verify approximate numerical results.
Although this method was originally developed by Vakakis (1993) to study intersections of stable and unstable manifolds for equations for Although this method was originally developed by Vakakis (1993) to study interseccould not be used, this method is applied herein because it is general enough to yield
exact solutions to general equations such as the multiple frequency forcing case being
studied. This multiple frequency forcing case is exact solutions to general equations such as the multiple frequency forcing case being exact solutions to general equations such as the multiple frequency forcing case being
studied. This multiple frequency forcing case is an engineering approximation used
in time domain simulations and physical scale model studied. This multiple frequency forcing case is an engineering approximation used
in time domain simulations and physical scale model testing in wave tanks to model
a truly random seaway which occurs in nature. However, e in time domain simulations and physical scale model testing in wave tanks to model
a truly random seaway which occurs in nature. However, each time history repre-
sents a single realization of the random seaway. In order f a truly random seaway which occurs in nature. However, each time history represents a single realization of the random seaway. In order for the results of such an investigation to be useful, multiple realizations must be c sents a single realization of the random seaway. In order for the results of such an investigation to be useful, multiple realizations must be considered. One must then obtain average and standard deviation values of the m investigation to be useful, multiple realizations must be considered. One must then obtain average and standard deviation values of the manifold locations. Whether or not the pseudo-random representation of the spectra is obtain average and standard deviation values of the manifold locations. Whether or
not the pseudo-random representation of the spectra is a valid random representa-
tion is an extensively debated subject in naval architec not the pseudo-random representation of the spectra is a valid random representa-
tion is an extensively debated subject in naval architecture and offshore engineering
(Chakrabarti 1989). Although, for finite N, this repr tion is an extensively debated subject in naval architecture and offshore engineering (Chakrabarti 1989). Although, for finite N, this representation does not satisfy the condition of a Gaussian seaway or response, it doe condition of a Gaussian seaway or response, it does so in the limit as N . Alternative representations of the random seaway do exist and include filter noise. However, this may not be easily applicable to the present pro aly applicable to
4. Results FS

4. Results
The results are for two vessels, which essentially span the entire size range of ships
and floating offshore platforms, either existing or planned. The first set of results The results are for two vessels, which essentially span the entire size range of ships
and floating offshore platforms, either existing or planned. The first set of results
is for a small fishing vessel which is probably o The results are for two vessels, which essentially span the entire size range of ships
and floating offshore platforms, either existing or planned. The first set of results
is for a small fishing vessel which is probably o and floating offshore platforms, either existing or planned. The first set of results is for a small fishing vessel which is probably one of the smallest vessels to venture away from safety of shore. The second set of resu

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SCIENCES** planned MOB. Alternatively, the MOB SBU, if built, will be the largest floating planned MOB. Alternatively, the MOB SBU, if built, will be the largest floating
vessel ever built. The two sets of results are shown for comparison since they exhibit
such qualitatively different types of behaviour. Even t vessel ever built. The two sets of results are shown for comparison since they exhibit such qualitatively different types of behaviour. Even though the seaway intensities vessel ever built. The two sets of results are shown for comparison since they exhibit
such qualitatively different types of behaviour. Even though the seaway intensities
are different, with the *Patti-B* exposed to mild such qualitatively different types of behaviour. Even though the seaway intensities are different, with the *Patti-B* exposed to mild seaways and the MOB exposed to more severe seaways, the vessels' relative sizes are so are different, with the $Patti-B$ of
more severe seaways, the vessels'
are still qualitatively different.
An indication of if the basin be severe seaways, the vessels' relative sizes are so vastly different that the results
a still qualitatively different.
An indication of if the basin boundaries will be simple and smooth or fractal
d complicated is determ

are still qualitatively different.
An indication of if the basin boundaries will be simple and smooth or fractal
and complicated is determined by whether the manifolds intersect or not. As a first
step in determining wheth An indication of if the basin boundaries will be simple and smooth or fractal
and complicated is determined by whether the manifolds intersect or not. As a first
step in determining whether or not this will occur for the p and complicated is determined by whether the manifolds intersect or not. As a first
step in determining whether or not this will occur for the pseudo-randomly forced
system, we determine solutions which lie in both the sta step in determining whether or not this will occur for the pseudo-randomly forced
system, we determine solutions which lie in both the stable and unstable manifolds.
After this is done, the distance between the two solutio system, we determine solutions which lie in both the stable and unstable manifolds.
After this is done, the distance between the two solutions can then be determined
and this will indicate whether or not a manifold interse After this is done, the distance between the two solutions can then be determined
and this will indicate whether or not a manifold intersection has occurred. When
the distance between the two manifolds goes to zero, the ma and this will indicate whether or not a manifold intersection has occurred. When
the distance between the two manifolds goes to zero, the manifolds become tangent
and this is a critical value of the forcing. Beyond the val the distance between the two manifolds goes to zero, the manifolds become tangent
and this is a critical value of the forcing. Beyond the value of forcing where the
manifolds become tangent, the manifolds intersect and th and this is a critical value of the forcing. Beyond the value of forcing where the manifolds become tangent, the manifolds intersect and the safe basin begins to erode.
This is exactly what the Melnikov function (Falzarano manifolds become tangent, the manifolds intersect and the safe basin begins to erode.
This is exactly what the Melnikov function (Falzarano *et al.* 1992) is used for and what is being developed herein is simply a more ge This is exactly what the Melnikov function (Falzarano *et al.* 1992) is used for and what is being developed herein is simply a more general alternative to the Melnikov approach. The method described has several potential what is being developed herein is simply a more general alternative to the Melnikov
approach. The method described has several potential benefits over the classical
Melnikov approach. These benefits enable us to (a) analys approach. The method described has several potential benefits over the classical
Melnikov approach. These benefits enable us to (a) analyse very general systems
for which the Melnikov method is not valid, (b) obtain highe Melnikov approach. These benefits enable us to (a) analyse very general systems
for which the Melnikov method is not valid, (b) obtain higher-order results, and
(c) develop a visual projection of the manifolds for single-d (c) develop a visual projection of the manifolds for single-degree-of-freedom systems.
(a) *Patti-B safe basin boundary projected phase plane*

The first set of results is for physical parameters representing the clam dredge *Patti-B* (Falzarano *et al.* 1992) in beam seas and rolling in various intensity PM sea spectra. As stated previously, the sea spectra are approximated by a finite but large The first set of results is for physical parameters representing the clam dredge $Patti-B$ (Falzarano *et al.* 1992) in beam seas and rolling in various intensity PM sea spectra. As stated previously, the sea spectra are appr Patti-B (Falzarano *et al.* 1992) in beam seas and rolling in various intensity PM sea
spectra. As stated previously, the sea spectra are approximated by a finite but large
number of periodic components. As can be seen, w spectra. As stated previously, the sea spectra are approximated by a finite but large
number of periodic components. As can be seen, when the wind speed is increased and
the seaway intensity increases, the vessel's dynamic number of periodic components. As can be seen, when the wind speed is increased and
the seaway intensity increases, the vessel's dynamics change qualitatively. The upper
and lower stable manifolds change from smooth curves the seaway intensity increases, the vessel's dynamics change qualitatively. The upper
and lower stable manifolds change from smooth curves similar to the unforced system
to rather complicated curves indicating the possibil and lower stable manifolds change from smooth curves similar to the unforced system
to rather complicated curves indicating the possibility of manifold intersections. The
size of the safe operating region of the vessel is to rather complicated curves indicating the possibility of manifold intersections. The
size of the safe operating region of the vessel is somewhat related to when these
manifolds intersect and become fractal or complicated size of the safe operating region of the vessel is somewhat related to when these
manifolds intersect and become fractal or complicated. Figure 2 shows moderate- and
large-amplitude sea spectra plotted against frequency fo manifolds intersect and become fractal or complicated. Figure 2 shows moderate- and
large-amplitude sea spectra plotted against frequency for a range of wind speeds. The
wind speed is the single parameter describing the se large-amplitude sea spectra plotted against frequency for a range of wind speeds. The
wind speed is the single parameter describing the seaway intensity. Results for time-
varying roll-motion solutions contained within the wind speed is the single parameter describing the seaway intensity. Results for time-
varying roll-motion solutions contained within the upper and lower stable manifolds
projected phase planes for these sea spectra are giv varying roll-motion solutions contained within the upper and lower stable manifolds
projected phase-planes for these sea spectra are given in figure 6. When looking
at these projected phase-plane results, it should be reco at these projected phase-plane results, it should be recognized that the solutions depicted represent a time evolution of a single trajectory and are not Poincaré time at these projected phase-plane results, it should be recognized that the solutions
depicted represent a time evolution of a single trajectory and are not Poincaré time
samplings of the manifold. This explains the wrapping depicted represent a time evolution of a single trajectory and are not Poincaré time
samplings of the manifold. This explains the wrapping around the fixed point. The
random oscillation occurs on the average at the zero cr \Box \Box \Box random oscillation occurs on the average at the zero crossing period while the solution \Box \Box is slowly evolving towards the fixed point.

(*b*) *MOB safe basin boundary projected phase plane*

The second set of results is for parameters representing the USNA generic MOB The second set of results is for parameters representing the USNA generic MOB
hull-form at transit draft in beam seas rolling in various intensity NATO sea spec-
tra. As stated previously, the sea spectra are approximated The second set of results is for parameters representing the USNA generic MOB
hull-form at transit draft in beam seas rolling in various intensity NATO sea spec-
tra. As stated previously, the sea spectra are approximated tra. As stated previously, the sea spectra are approximated by a finite number of *Phil. Trans. R. Soc. Lond.* A (2000)

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Figure 6. (a) *Patti-B* projected phase plane for PM spectra $(U_w = 2.75 \text{ m s}^{-1})$. (b) *Patti-B*
projected phase plane for PM spectra $(U_w = 5.15 \text{ m s}^{-1})$. (c) *Patti-B* projected phase plane for Figure 6. (a) Patti-B projected phase plane for PM spectra $(U_w = 2.75 \text{ m s}^{-1})$. (b) Patti-B projected phase plane for PM spectra $(U_w = 5.15 \text{ m s}^{-1})$. (c) Patti-B projected phase plane for PM spectra $(U = 10.0 \text{ m s}^{-1})$ projected phase plane for PM spectra $(U_{\rm w} = 5.15 \text{ m s}^{-1})$. (c) *Patti-B* projected phase plane for PM spectra $(U_{\rm w} = 10.0 \text{ m s}^{-1})$.

periodic components. As can be seen, when the seaway intensity increases, the vesperiodic components. As can be seen, when the seaway intensity increases, the vessel's dynamics change qualitatively. The distance between the upper and lower stable manifolds and the roll axis in the neighbourhood of the periodic components. As can be seen, when the seaway intensity increases, the vessel's dynamics change qualitatively. The distance between the upper and lower stable manifolds and the roll axis in the neighbourhood of the sel's dynamics change qualitatively. The distance between the upper and lower stable
manifolds and the roll axis in the neighbourhood of the angle of vanishing stability
changes as the intensity of the seaway increases. As manifolds and the roll axis in the neighbourhood of the angle of vanishing stability
changes as the intensity of the seaway increases. As the wave amplitude increases,
the magnitude of the unstable periodic orbit in the ne changes as the intensity of the seaway increases. As the wave amplitude increases, the magnitude of the unstable periodic orbit in the neighbourhood of the angle of vanishing stability increases. In our previous analysis o the magnitude of the unstable periodic orbit in the neighbourhood of the angle of vanishing stability increases. In our previous analysis of a small fishing vessel above, the manifolds changed from smooth curves similar to complicated curves, indicating the possible presence of manifold intersections. Since the MOB is so large, these rather severe seaways do not dramatically affect the manifolds. Figure 3b; c shows moderate- and large-amplitude Bretschneider sea spectra plot-
Figure 3b, c shows moderate- and large-amplitude Bretschneider sea spectra plot-
d versus frequency for sea-state intensities 5 and 9 with t

manifolds.
Figure $3b$, c shows moderate- and large-amplitude Bretschneider sea spectra plot-
ted versus frequency for sea-state intensities 5 and 9, with the sea-state intensity
heing the variable parameter. The projec being the variable parameter. The projected phase planes results for time-varying being the variable parameter. The projected phase planes results for time-varying roll-motion solutions contained within the upper and lowe roll-motion solutions contained within the upper and 9, with the sea-state intensity
being the variable parameter. The projected phase planes results for time-varying
roll-motion solutions contained within the upper and lo being the variable parameter. The projected phase planes results for time-varying roll-motion solutions contained within the upper and lower stable manifolds are given in figure 7. It should be noted that the results give in figure 7. It should be noted that the results given are non-dimensionalized, using the non-dimensionalization implied by (3.3) .

(*c*) *Patti-B extended state-space results*

Once unstable manifolds are included, the two-dimensional projection of the time-Once unstable manifolds are included, the two-dimensional projection of the time-
varying solutions which lie in the stable or unstable manifolds may be deceptive. This
is so because true intersections only occur for the Once unstable manifolds are included, the two-dimensional projection of the time-
varying solutions which lie in the stable or unstable manifolds may be deceptive. This
is so because true intersections only occur for the s *Phil. Trans. R. Soc. Lond.* A (2000)

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the 7. MOB projected phase plane for NATO sea spectra: (*a*) sea state (for $[H_s, T_0] = [10.7, 9.7]$); (*b*) sea state 9 (for $[H_s, T_0] = [45.9, 20.0]$).

Figure 8. *Patti-B* extended phase space showing solutions contained in stable and unstable nded phase space showing solutions contained in
manifold $(U_{\rm w} = 2.75 \text{ m s}^{-1}, V = 2.75 \text{ m s}^{-1}).$

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Downloaded from rsta.royalsocietypublishing.org*A new meth[od to predict vessel/platf](http://rsta.royalsocietypublishing.org/)orm dynamics* ¹⁹⁷⁹ **MATHEMATICAL,
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& ENGINEERING
SCIENCES** $5\angle$ $w^{-s}(t)$ 0 \angle THE ROYAL *t* $+u_{f}$ ₁ $w^{+u}(t)$ (t) (*b*) SOCI $-5\angle$ **PHILOSOPHICAL**
TRANSACTIONS 2 ŏ 1 $\phi(t)$ ⁰ $\left\{ \begin{array}{c} 0 \\ 0 \end{array} \right\}$ -1 \vdash $^{-2}$ -2 **MATHEMATICAL,
PHYSICAL
& ENGINEERING
SCIENCES** -1 0 ⁻ $\phi(t)$ ¹ 2 Figure 8. (*Cont.*)

dimensional extended state-space representation is the only unique representation. In
order to illustrate this, and in order to determine whether or not intersections have dimensional extended state-space representation is the only unique representation. In order to illustrate this, and in order to determine whether or not intersections have occurred some typical results for the *Patti-B* a dimensional extended state-space representation is the only unique representation. In order to illustrate this, and in order to determine whether or not intersections have occurred, some typical results for the *Patti-B* a order to illustrate this, and in order to determine whether or not intersections have occurred, some typical results for the *Patti-B* are provided. These one-dimensional solution curves are displayed in the full three-di occurred, some typical results for the *Patti-B* are provided. These one-dimensional solution curves are displayed in the full three-dimensional extended state space (figure 8). These results clearly indicate that the two solution curves are displayed in the full three-dimensional extended state space (figure 8). These results clearly indicate that the two curves do not intersect for the two given seaway intensities. A more extensive and sy ure 8). These results clearly indicate that the two curves do not intersect for the
two given seaway intensities. A more extensive and systematic investigation is cur-
rently underway. In order to determine more clearly if two given seaway intensities. A more extensive and systematic investigation is currently underway. In order to determine more clearly if the manifolds' intersections have occurred, the entire manifold must be generated. Ge fold would involve varying the initial time to and then generated. Generating the entire manifold would involve varying the initial time t_0 and then generating an entire manifold mesh. After this had been done, the dis fold would involve varying the initial time t_0 and then generating an entire manifold mesh. After this had been done, the distance between the two manifolds, i.e. stable and unstable, can then be calculated. This dista fold would involve varying the initial time t_0 and then generating an entire manifold
mesh. After this had been done, the distance between the two manifolds, i.e. stable
and unstable, can then be calculated. This dista mesh. After this had been done, the distance between the two manifolds, i.e. stable
and unstable, can then be calculated. This distance going to zero would indicate
that manifold intersections were imminent. This would be and unstable, can then be calculated. This distance going to zero would indicate that manifold intersections were imminent. This would be a critical value of external wave forcing since, at a greater value of wave forcing, erode.

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5. Conclusions
The described method is quite powerful and capable of handling very general systems. The described method is quite powerful and capable of handling very general systems.
The application herein used the knowledge of the zeroth-order solution, which was
known in closed form for this simple system. However, t The described method is quite powerful and capable of handling very general systems.
The application herein used the knowledge of the zeroth-order solution, which was
known in closed form for this simple system. However, t The application herein used the knowledge of the zeroth-order solution, which was
known in closed form for this simple system. However, this is not a requirement and
actually for more general systems it could be known nume known in closed form for this simple system. However, this is not a requirement and actually for more general systems it could be known numerically. Clearly, the safe operating region of the vessel is directly related to w operating region of the vessel is directly related to when the calculated stable and unstable manifolds intersect and erode the safe basin. It should be re-emphasized here operating region of the vessel is directly related to when the calculated stable and
unstable manifolds intersect and erode the safe basin. It should be re-emphasized here
that the results given correspond to single realiz unstable manifolds intersect and erode the safe basin. It should be re-emphasized here
that the results given correspond to single realizations of the given sea spectra. In
order to gain a more complete probabilistic under that the results given correspond to single realizations of the given sea spectra. In order to gain a more complete probabilistic understanding of the system's random behaviour, multiple realizations must be considered and order to gain a more complete probabilistic understanding of the system's random
behaviour, multiple realizations must be considered and analysed. This ensemble
of results should then be analysed in terms of averages and s behaviour, multiple realizations must be considered and anal
of results should then be analysed in terms of averages and
However, this has not yet been done in a systematic manner.
The results clearly demonstrate the effec results should then be analysed in terms of averages and standard deviations.
wever, this has not yet been done in a systematic manner.
The results clearly demonstrate the effect of random excitation on the global non-
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However, this has not yet been done in a systematic manner.
The results clearly demonstrate the effect of random excitation on the global non-
linear dynamics of both vessels about their roll axes. However, it should be no The results clearly demonstrate the effect of random excitation on the global non-
linear dynamics of both vessels about their roll axes. However, it should be noted
again that, in general, for typical semi-submersibles, linear dynamics of both vessels about their roll axes. However, it should be noted again that, in general, for typical semi-submersibles, the roll axis is not the critical rotation axis. The critical axis is often an appro rotation axis. The critical axis is often an approximately diagonal axis (Kota et 1998). It is believed that due to the large length-to-beam ratio of the MOB SBU the roll axis is most probably the critical axis or quite

the roll axis is most probably the critical axis or quite close to the critical axis.
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for their past and present support of this and re the support of the US National Science Foundation, Dynamical Systems and Control Program

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